



## Resumos

### Abstracts

Sessão: Equações Diferenciais Parciais

*Session: Partial Differential Equations*

### Organizadores

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# A desigualdade de Gagliardo-Nirenberg e aplicações

Ademir Pastor\*

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## Resumo

Recordaremos o papel essencial que a desigualdade de Gagliardo-Nirenberg possui no estudo de boa colocação global de algumas equações de evolução. Também, veremos que quando trabalhamos com equações críticas ou supercríticas, a melhor constante aparecendo na referida desigualdade possui fundamental importância. Por fim, apresentaremos uma nova desigualdade do tipo Gagliardo-Nirenberg anisotrópica bem como sua melhor constante.

# On the self-similar blow-up scenario for the Euler equations

Anne Bronzi\*

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## Resumo

In this talk we will survey some results regarding the possibility of a self-similar blow-up for the Euler equations. We will also prove that under a mild  $L^p$ -growth assumption on the self-similar profile we obtain that the solution carries a positive amount of energy up to the time of blow-up. As a consequence, we will recover and extend several previously known exclusion criteria. Also, we will present some preliminary studies on the fractal dimension of the energy measure, which roughly speaking is the limit of the measures on the space induced by the velocity squared as time approaches the time of blow-up. We will explore the relation between the fractal dimension of the energy measure and the growth of the velocity as time approaches the time of singularity formation. This is joint work with Roman Shvydkoy.

# Regularity theory for time dependent mean-field games

Edgard Pimentel\*

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## Resumo

In this talk, we present a series of results on the existence of smooth solutions for time dependent mean-field games systems (MFG, for short). First, we consider a power-like dependence on the measure and Hamiltonians satisfying both sub and super-quadratic growth conditions. In the sub-quadratic case, this is achieved by combining a Gagliardo-Nirenberg type of argument with a new class of polynomial estimates for solutions of the Fokker-Planck. For the super-quadratic setting, we recur to a delicate argument based on the non-linear adjoint method. In the sequel, we investigate logarithmic couplings, which introduce several mathematical challenges. In order to by-pass these difficulties, we recur to the concavity properties of the logarithmic function, combined with the structure of the Fokker-Planck equation. This yields uniform estimates for the coupling in suitable Lebesgue spaces. These build upon the Lipschitz regularity for the Hamilton-Jacobi equation to produce classical solutions under certain restrictions on the growth regime of the Hamiltonian. Minor extensions and general ideas on applications of our results close the talk. This is based on a series of joint works with D. Gomes and H. Sánchez-Morgado.

# Existence results for the fractional Choquard equation

Gaetano Siciliano\*

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## Resumo

In the talk we present recent results on the following system involving the fractional Laplacian in the whole  $\mathbb{R}^N$ ,  $N \geq 3$ :

$$(-\Delta)^s u + u = (\mathcal{K}_\alpha \star |u|^p)|u|^{p-2}u.$$

Here,  $\mathcal{K}_\alpha = |x|^{\alpha-N}$ ,  $s \in (0, 1)$ ,  $\alpha \in (0, N)$  and  $p$  varies in a suitable range. We look for solutions  $u \in H^s(\mathbb{R}^N)$  by using variational methods; indeed weak solutions (which are shown to be regular) to the above problem can be characterized as critical points of a functional. By exploiting the property of this functional, positive ground state and also infinitely many (possibly changing sign) solutions are found. These results are obtained in collaboration with P. d'Avenia (Politecnico di Bari, IT) and M. Squassina (Univ. di Verona, IT).

# Singular perturbation methods for fully nonlinear degenerate equations: a geometric approach

Gleydson Chaves Ricarte\*

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## Resumo

We establish new, optimal gradient continuity estimates for solutions to a class of 2nd order partial differential equations,

$$|\nabla u|^\gamma F(X, D^2u) = f,$$

whose diffusion properties (ellipticity) degenerate along the *a priori* unknown singular set of an existing solution,  $S(u) := \{X : \nabla u(X) = 0\}$ . The innovative feature of our main result concerns its optimality – the sharp, encoded smoothness effects of the operator. Such a quantitative information usually plays a decisive role in the analysis of a number of analytic and geometric problems.

For the second part, we study solutions of one phase singular degenerate singular perturbation problems of the type:

$$|\nabla u|^\gamma F(X, D^2u) = \beta_\epsilon(u),$$

where  $\beta_\epsilon$  approaches Dirac  $\delta_0$  as  $\epsilon \rightarrow 0$ . Uniform local Lipschitz regularity is obtained for these solutions. The existence theory for non variational (least supersolutions) solutions for this problem is developed. Uniform linear growth rate with respect to the distance from the  $\epsilon$ -level surfaces are established for this nonvariational solutions. Finally, letting  $\epsilon \rightarrow 0$  basic properties such as local Lipschitz regularity and non-degeneracy property are proven for the limit and a Hausdorff measure estimate for its free boundary is obtained. An important question on modern theory of elliptic free boundaries problems concerns whether is possible extend the local optimal regularity of singularly perturbed problems up to the boundary. In this article, under appropriate conditions on the operator  $F$  we study regularity up to the boundary for one-phase singularly perturbed fully nonlinear elliptic problems,

$$F(X, \nabla u^\epsilon, D^2u^\epsilon) = \beta_\epsilon(u^\epsilon), \quad \text{in } \Omega$$

we establish global gradient bounds independent of the parameter  $\epsilon$ .

# Estimativas de Sobolev nos espaços de Hardy locais

Gustavo Hoepfner\*

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## Resumo

Nesta palestra apresentaremos condições suficientes para a continuidade de operadores pseudo-diferenciais (com símbolos de ordem negativa) nos espaços de Hardy locais.

# Existence of maximizers for Airy-Strichartz inequalities

Luiz Gustavo Farah\*

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## Resumo

Recently, in a joint work with Ademir Pastor, we give a simple proof of the classical well-posedness result for the generalized KdV equation proved by Kenig, Ponce and Vega. The key ingredient in the proof is the following Airy-Strichartz estimate

$$\|U(t)u_0\|_{L_x^{5k/4}L_t^{5k/2}} \leq C_k \|u_0\|_{\dot{H}_x^{s_k}},$$

where  $k > 4$ ,  $s_k = (k-4)/2k$  and  $U(t)$  denotes the linear propagator for the KdV equation. Our goal here is to prove the existence of maximizers for the above inequality. The main tool we use is a linear profile decomposition for the Airy equation with initial data in  $\dot{H}_x^{s_k}$ . As a consequence, we also establish the existence of maximizers for a general class of Strichartz type inequalities associated to the generalized KdV equation.

# Problema de Transmissão com Fronteira Livre

Marcelo Amaral\*

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## Resumo

Nós estudamos problemas de transmissão com fronteira livre de um meio aleatório para outro. Exigi-se que as soluções resolvam distintas equações diferenciais parciais,  $L_+$  e  $L_-$ , nos seus conjuntos de positividade e negatividade, respectivamente. Um correspondente fluxo de balanço de uma fase para outra também é imposto. Estabelecemos existência e limitação  $L^\infty$  das soluções. Também provamos que soluções variacionais são não-degeneradas e desenvolvemos a teoria de regularidade para soluções de tais problemas de fronteira livre.

# The Neumann problem in oscillating thin domains

Marcone Pereira\*

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## Resumo

In this talk we discuss some recent results about the asymptotic behavior of the solutions of a semilinear parabolic equation with homogeneous Neumann boundary conditions posed on thin domains with locally periodic structure on the boundary. We discuss the limit problem dependence with respect to the geometry of the thin domain and its oscillating boundary also studying the convergence of the global attractors.

# Existência e concentração em esferas de soluções de equações de Schrödinger não-lineares

Marcos Pimenta\*

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## Resumo

Nesse trabalho estudamos questões de existência de soluções nodais e a presença do fenômeno de concentração em esferas para a versão estacionária de uma equação de Schrödinger não-linear. O potencial considerado satisfaz uma condição local e a não-linearidade é do tipo potência superlinear e subcrítica. Métodos variacionais são utilizados juntamente com uma análise do funcional energia sobre a variedade de Nehari associada ao problema.

# New developments in the lower dimensional obstacle problem

Mariana Smit Vega Garcia\*

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## Resumo

We will describe the Signorini, or lower-dimensional obstacle problem, for a uniformly elliptic, divergence form operator  $L = \operatorname{div}(A(x)\nabla)$  with Lipschitz continuous coefficients. We will give an overview of what is known about this problem, new developments and the role of a new monotonicity formula for an appropriate generalization of Almgren's frequency functional in the optimal regularity of the solution. Similarly to what happens when  $L = \Delta$ , one of our main results states that the variational solution has the optimal interior regularity  $C_{loc}^{1, \frac{1}{2}}(\Omega_{\pm} \cup \mathcal{M})$ , when  $\mathcal{M}$  is a codimension one flat manifold which supports the obstacle and divides the domain  $\Omega$  into two parts,  $\Omega_+$  and  $\Omega_-$ . This is joint work with Nicola Garofalo.

# A multiplicity result for the Chern-Simons-Schrödinger equation with a general nonlinearity

Patrícia Leal da Cunha\*

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## Resumo

We give a multiplicity result for the following Chern-Simons-Schrödinger equation

$$-\Delta u + 2qu \int_{|x|}^{\infty} \frac{u^2(s)}{s} h_u(s) ds + qu \frac{h_u^2(|x|)}{|x|^2} = g(u), \quad \text{in } \mathbb{R}^2,$$

where  $h_u(s) = \int_0^s \tau u^2(\tau) d\tau$ , under very general assumptions on the nonlinearity  $g$ . In particular, for every  $\ell \in \mathbb{N}$ , we prove the existence of (at least)  $\ell$  distinct solutions, for every  $q \in (0, q_\ell)$  and a suitable  $q_\ell$ .

# Regularidade para Equações Integro-diferenciais Totalmente Não-lineares Anisotrópicas

Raimundo Leitão\*

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## Resumo

Consideramos equações integro-diferenciais não-lineares governadas por núcleos que possuem homogeneidades diferentes em diferentes direções. Provamos uma versão não-local da estimativa ABP, uma desigualdade de Harnack e a regularidade  $C^{1,\gamma}$  estendendo os resultados de Caffarelli e Silvestre (Comm Pure Appl Math 62:597-638, 2009) para o caso anisotrópico.

# Boltzmann Model for Viscoelastic Particles

Ricardo Alonso\*

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## Resumo

In this talk we investigate the long-time behavior of a system of viscoelastic particles modeled with the homogeneous Boltzmann equation and prove the existence of a universal Maxwellian intermediate asymptotic state with explicit rate of convergence towards it. Exponential lower pointwise bounds and propagation of regularity are also studied. These results can be seen as a generalization of several classical results holding for the pseudo-Maxwellian and constant normal restitution models.

# A positive mass theorem for asymptotically flat manifolds with a non-compact boundary

Sérgio Almaraz\*

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## Resumo

I will discuss a positive mass theorem for non-compact manifolds with boundary, which have ends asymptotic to the Euclidean half-space. For spin manifolds or for dimensions up to 7, our result settles a conjecture posed in my recent paper concerning the convergence of a Yamabetype flow on manifolds with boundary. This is a joint work with Ezequiel Barbosa (UFMG) and Levi de Lima (UFC).