Characterization of linearly Lindelöf topological spaces through family of discrete sets

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Resumo

Let (X, τ) be a topological space, we say that X is linearly Lindelöf, if every open and increasing covering of X admits a countable subcover. Let $\mathscr{A}$ be the family of discrete subsets of X. For any $A \in \mathscr{A}$, we denote: $A^{\perp} = \{x \in X \setminus A : A \cup \{x\} \notin \mathscr{A}\}$. If $A^{\perp} \neq \emptyset$ we can choose a discrete $\emptyset \neq A_1 \subset A^{\perp}$, in general if $\alpha = \theta + 1$ is a successor ordinal and $A_{\theta}^{\perp} \neq \emptyset$ we can choose a discrete $\emptyset \neq A_{\alpha} \subset A_{\theta}^{\perp}$, if κ is a limit ordinal and $\bigcap_{\alpha<\kappa} A_{\alpha}^{\perp} \neq \emptyset$ we can choose a discrete $\emptyset \neq A_{\kappa} \subset \bigcap_{\alpha<\kappa} A_{\alpha}^{\perp}$. If we continue this procedure until an ordinal μ, we have a discrete chain starting at A: $C_A = \{A_{\kappa} : \kappa < \mu\}$. We say that C_A collapses if $\bigcap_{\kappa<\mu} A_{\kappa}^{\perp} = \emptyset$, we also say that μ is the length of the chain. For all well ordered discrete sets $D = \{d_{\alpha} : \alpha < \theta\}$ with $cf(\theta) \geq \omega_1$, we denote $D_{\gamma} = \{d_{\alpha} : \gamma \leq \alpha < \theta\}$, for all $\gamma < \theta$. In this work we characterize linearly Lindelöf topological spaces, via discrete chains, as follows: let X be a topological space T_1, then X is linearly Lindelöf if, and only if, all discrete chain such that the cofinality of its length is greater or equal than ω_1 does not collapse and for all well ordered discrete sets $D = \{d_{\alpha} : \alpha < \theta\}$ with $cf(\theta) \geq \omega_1$, we have $\bigcap_{\gamma<\theta} D_{\gamma}^{\perp} = \emptyset$.

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