Block designs from algebraic point of view

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Resumo

Steiner triple systems play a major part in combinatorics; many interesting connections have been developed between their combinatorial and algebraic aspects. From this point of view, the study of their algebraic background can be useful.

This generates an interest towards Steiner quasigroups and loops. In this presentation we analyse their multiplication and automorphism groups. Specifically, we discuss which groups can be multiplication groups of Steiner loops (this concept is important for non-associative structures). This question has been solved for several classes of Steiner quasigroups and loops. For example, we prove that all automorphisms of a free Steiner loop (FSL) are tame, and the automorphism group cannot be finitely generated when the loop has more than 3 generators.

The automorphism group of the 3-generated FSL is generated by the symmetric group S_3 and by the elementary automorphism $\varphi = e_1(x_2)$. We also conjecture that $Aut(S(x_1, x_2, x_3))$ is the Coxeter group $< (12), (13), \varphi | (\varphi(12))^3 = (\varphi(13))^4 = ((12)(13))^3 = 1 >$. These conjecture fits the context of the work by U. Umirbaev on linear Nielsen-Schreier varieties of algebras.

Furthermore, we discuss their growth and Cayley graphs.