

# On locally nilpotent derivations of Fermat Rings

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## Resumo

Let  $\mathbb{C}[X_1, \dots, X_n]$  be the polynomial ring in  $n$  variables over complex numbers  $\mathbb{C}$ . Define

$$B_n^m = \frac{\mathbb{C}[X_1, \dots, X_n]}{(X_1^m + \dots + X_n^m)},$$

where  $m \geq 2$  and  $n \geq 3$ . This ring is known as Fermat ring.

In a recent paper [4] D. Fiston and S. Maubach show that for  $m \geq n^2 - 2n$  the unique locally nilpotent derivation of  $B_n^m$  is the zero derivation. Consequently the following question naturally arises: is the unique locally nilpotent derivation of the Fermat ring  $B_n^m$  for  $m \geq 2$  and  $n \geq 3$  the zero derivation?

In the paper [1] we show that the answer to this question is negative for  $m = 2$  and  $n \geq 3$ . In other words, there exist locally nilpotent derivations over  $B_n^2$  nontrivial. Furthermore, we show that these derivations are irreducible. In the general case, we prove that for certain classes of derivations of  $B_n^m$  the unique locally nilpotent derivation is the zero derivation.

The question remains open for the case  $m \geq 3$ .

## Referências

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