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Basic superranks for varieties of algebras

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Resumo

In the present work, all algebras are considered over a field of characteristic 0. Let $\mathcal{V}$ be a variety of algebras and $\mathcal{V}_r$ be a subvariety of $\mathcal{V}$ generated by the free $\mathcal{V}$-algebra of rank r. Then one can consider the chain $\mathcal{V}_1 \subseteq \mathcal{V}_2 \subseteq \cdots \subseteq \mathcal{V}_r \subseteq \cdots \subseteq \mathcal{V}$, where $\mathcal{V} = \bigcup_r \mathcal{V}_r$. If this chain stabilizes, then the minimal number r with the property $\mathcal{V}_r = \mathcal{V}$ is called the *basic rank* of the variety $\mathcal{V}$ and is denoted by $r_b(\mathcal{V})$. Otherwise, we say that $\mathcal{V}$ has the *infinite basic rank* $r_b(\mathcal{V}) = \aleph_0$.

Recall the main results on the basic ranks of the varieties of associative (Assoc), Lie (Lie), alternative (Alt), Malcev (Malc), and some other algebras. It was first shown by A. I. Mal'cev [1] that $r_b(\text{Assoc}) = 2$. A. I. Shirshov [2] proved that $r_b(\text{Lie}) = 2$ and $r_b(\text{SJord}) = 2$, where SJord is the variety generated by all special Jordan algebras. In 1958, A. I. Shirshov posed a problem on finding basic ranks for alternative and some other varieties of nearly associative algebras [3, Problem 1.159]. In 1977, I. P. Shestakov proved that $r_b(\text{Alt}) = r_b(\text{Malc}) = \aleph_0$ [2, 4]. The similar fact for the variety of algebras of type $(-1, 1)$ was established by S. V. Pchelintsev [5]. Note that the basic ranks of the varieties of Jordan and right alternative algebras are still unknown.

A proper subvarietiy of associative algebras can be of infinite basic rank as well. For instance, so is the variety $\mathrm{V}ar\,\mathrm{G}$ generated by the Grassmann algebra G on infinite number of generators, or the variety defined by the identity $[x, y]^n = 0,\ n > 1$.

It follows from the Kemer's Theorem [6] that the ideal of identities of arbitrary associative algebra coincides with the ideal of identities of the Grassmann envelope [7] of some finite dimensional superalgebra. This result suggests a generalization of the notion of basic rank that we call basic superrank.

First we consider a number of varieties of nearly associative algebras that have infinite basic ranks and calculate their basic superranks which turns out to be finite. Namely we prove that the variety of alternative metabelian (solvable of index 2) algebras has the two

basic superranks $(1, 1)$ and $(0, 3)$; the varieties of Jordan and Malcev metabelian algebras have the unique basic superranks $(0, 2)$ and $(1, 1)$, respectively. Furthermore, for arbitrary pair $(r, s) \neq (0, 0)$ of nonnegative integers we provide a variety that has the unique basic superrank (r, s). Finally, we construct some examples of nearly associative varieties that do not possess finite basic superranks.

This is a joint work with Ivan Shestakov.

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