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Construction of lattices from quaternion algebras

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Resumo

New advances in wireless communications consider systems with multiple antennas at both the transmitter and receiver ends, in order to increase the data rates and the reliability. The coding problem then becomes more complex and code design criteria for such scenarios showed that the challenge was to construct fully-diverse, full-rate codes, i.e., sets of matrices such that the difference of any two distinct matrices is full rank. This requires new algebraic tools, namely division algebras. Division algebras are non-commutative algebras that naturally yield families of fully-diverse codes, thus enabling to design high rate, highly reliable Space-Time codes [1]. Space-Time Codes based on an order of a quaternion algebra such that the volume of the Dirichlet's polyhedron of the group of units is small, are better suited for decoding using the method of algebraic reduction since the approximation error is smaller [2]. The volume of this Dirichlet's polyhedron is given by the Tamagawa formula and is called the Tamagawa volume [3]. In this work we propose to construct the E_8-lattice as a left ideal of a maximal order of some quaternion algebras with a small Tamagawa volume.

Referências

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