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Compact gradient generalized quasi-Einstein metrics with constant scalar curvature

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Resumo

A gradient generalized m-quasi-Einstein metric on a complete Riemannian manifold (M^n, g) is a choice of a potential function $f : M^n \to \mathbb{R}$ as well as a function $\lambda : M^n \to \mathbb{R}$ such that

$$Ric + \nabla^2 f - \frac{1}{m} df \otimes df = \lambda g, \tag{1}$$

where Ric denotes the Ricci tensor of $(M^n,\, g)$, while $0 < m \leq \infty$ is an integer, ∇^2 and $\otimes$ stand for the Hessian and the tensorial product, respectively.

It is important to point out that if $m = \infty$ and λ is constant, equation (1) reduces to one associated to a gradient Ricci soliton, as well as considering $m = \infty$ and λ not constant we obtain the almost Ricci soliton equation. In addition, if λ is constant and m is a positive integer, it corresponds to m-quasi-Einstein metrics that are exactly those n-dimensional manifolds which are the base of an $(n + m)$-dimensional Einstein warped product. He, Petersen and Wylie was given some classification for m-quasi-Einstein metrics where the base has non-empty boundary. Moreover, they have proved a characterization for m-quasi-Einstein metric when the base is locally conformally flat. We also point out that, Catino have proved that around any regular point of f a generalized m-quasi Einstein metric $\big(M^n,\, g,\, \nabla f,\, \lambda\big)$ with harmonic Weyl tensor and $W(\nabla f, \cdots, \nabla f) = 0$ is locally a warped product with $(n-1)$-dimensional Einstein fibers.

In this lecture we shall show that a compact gradient generalized m-quasi-Einstein metric $\big(M^n,\, g,\, \nabla f,\, \lambda\big)$ with constant scalar curvature must be isometric to a standard Euclidean sphere $\mathbb{S}^n$ with the potential f well determined. This is a joint work with Abdênago Barros (UFC-CE).