On Reproducing Kernel Hilbert Spaces and Approximation

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Resumo

Let X be a locally compact topological space and $K : X \times X \to \mathbb{C}$ be a continuous (hermitian) kernel. We are mainly concerned with integral operators $\mathcal{K} : L^2(X, \mu) \to L^2(X, \mu)$ of the form

$$\mathcal{K}(f)(x) := \int_X K(x, y) f(y) \, d\mu(y), \quad f \in L^2(X, \mu), \quad x \in X,$$

which are positive in the sense that

$$\langle \mathcal{K}(f), f \rangle_{L^2} \geq 0, \quad f \in L^2(X, \mu).$$

If μ is a strictly positive Borel measure, then this setting implies that this kernel is positive definite in the usual sense ([1]), that is,

$$\sum_{i,j=1}^{n} \overline{c_i} c_j K(x_i, x_j) \geq 0,$$

for all $n \geq 1$, $x_1, x_2, \ldots, x_n \in X$ and $c_1, c_2, \ldots, c_n \in \mathbb{C}$. This condition enable us to define an inner product on the (reproducing kernel) Hilbert space $\mathcal{H}_K$ containing $\{K^x := K(\cdot, x) : x \in X\}$ as subset, where

$$\langle K^x, K^y \rangle_K := K(y, x), \quad x, y \in X,$$

and holds the *reproducing property*

$$f(x) = \langle f, K^x \rangle_K, \quad f \in \mathcal{H}_K, \quad x \in X.$$

Among other things, this property ensures that $\mathcal{H}_K$ is composed of continuous functions only ([2]). This means that $\mathcal{H}_K$ is a subset of $C(X)$ and sometimes of $L^2(X, \nu)$. We may then ask in some applications: Is it a dense subset of one of those spaces? ([2, 3]).

To finish this abstract we would like to say that the Hilbert space structure of $\mathcal{H}_K$ and its relation to (positive) integral operators enter in the solution of many problems. Among this problems, we are now interested in approximate solutions of some integral equations [4] and analyze some density problems, trying to contribute in some lines we referred to.

Referências

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