

Twisted Hall algebra of bound quiver with small homological dimension

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Resumo

Let Q be a quiver, \mathfrak{g} be a symmetric Kac-Moody algebra associated with Q , and $\text{Rep}_Q(\mathbb{F}_q)$ be the category of finite-dimensional representations of Q over a field \mathbb{F}_q . In his remarkable papers [1, 2] C. Ringel proved that if Q is a Dynkin quiver then there exists an isomorphism between the Hall algebra associated with $\text{Rep}_Q(\mathbb{F}_q)$ and the positive part of quantized universal enveloping algebra $U_t(\mathfrak{g})$ with $t^2 = q$.

We consider the case of a bound quiver Q assuming that the global dimension of Q is at most 2. To each such quiver we associate an associative algebra $U_q(Q)$ given by relations and generators. In the case when Q is a representation-directed we show that there exists an isomorphism between $U_q(Q)$ and the corresponding twisted Hall algebra $\mathbf{H}_{\text{Rep}_Q(\mathbb{F}_q)}^{tv}$.

As the limiting case of this construction we also study representations of commutative quivers over the so-called field with one element: \mathbb{F}_1 . Such a field is not defined per se, but there is agreement on what should be the definition and basic properties of the category of vector spaces over \mathbb{F}_1 as a limiting case of the categories of vector spaces over \mathbb{F}_q (see for example [3]). Following the ideas of M. Szczesny we show that the category $\text{Rep}_Q(\mathbb{F}_1)$ has enough structure to define its Hall algebra and prove that there exists an epimorphism $\rho : U_1(Q) \rightarrow \mathbf{H}_{\text{Rep}_Q(\mathbb{F}_1)}$.

Based on joint work [4] with Evan Wilson.

Referências

- [1] Ringel, C. M., *From representations of quivers via Hall and Loewy algebras to quantum groups*, Proceedings Novosibirsk Conference 1989, Contemporary Mathematics 131, (Part 2), 381–401 (1992).

- [2] Ringel, C. M., *Hall algebras and quantum groups*, Invent. Math. **101**, no. 3, 583–591, (1990).
- [3] Szczesny, M., *Representations of Quivers Over \mathbb{F}_1 and Hall Algebras*, International Mathematics Research Notices, no. 10, 2377–2404 (2012).
- [4] Evan Wilson, Kostiantyn Iusenko, *Some remarks on Hall algebra of bound quiver*. to be published.