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Phase transition in ferromagnetic Ising model with a cell-board external field

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Abstract

In this paper we show the presence of a first-order phase transition for a ferromagnetic Ising model on $\mathbb{Z}^2$ with a periodical external magnetic field (proposed by Maruani et al. [4]). The external field takes two values $\pm h$, with $h > 0$, composing a cell-board configuration with rectangular cells of sides $L_1 \times L_2$ sites, such that total value of the external field is zero. Formally, for each n, m integers we define

$$C(n,m) = \{(t_1,t_2) \in \mathbb{Z}^2 : \quad nL_1 \le t_1 < (n+1)L_1, \\ mL_2 \le t_2 < (m+1)L_2\},$$

then

$$\mathbf{Z}_+ = \bigcup_{\substack{n,m: \\ n+m \text{ is even}}} C(n,m), \qquad \mathbf{Z}_- = \mathbb{Z}^2 \setminus \mathbf{Z}_+.$$

Let $\sigma \in \Omega = \{-1,+1\}^{\mathbb{Z}^2}$ be a configuration on $\mathbb{Z}^2$. We study the model with a formal Hamiltonian defined for any $\sigma \in \Omega$ as

$$H(\sigma) = -J \sum_{\langle t,s \rangle} \sigma(t)\sigma(s) - \sum_s h_s \sigma(s),$$

where $J > 0$, the symbol $\langle t,s \rangle$ denotes nearest neighbours $s,t \in \mathbb{Z}^2$, that is $|t-s| = 1$, and

$$h_s = \begin{cases} h, & \text{if } s \in \mathbf{Z}_+, \\ -h, & \text{if } s \in \mathbf{Z}_-. \end{cases}$$

The phase transition holds if $h < \frac{2J}{L_1} + \frac{2J}{L_2}$. Our result is related with Nardi et al. [5], we prove a phase transition in a more general

model that the considered by them. Moreover, the phase transition in antiferromagnetic Ising model with constant external field (see [2]) holds true as a corollary of our proof.

We use an approach based on the technique of reflection positivity [1]. Particularly, we apply a certain key inequality which is usually referred to as the chessboard estimate. This tool allows us to construct a sort of the Peierls arguments evaluating the contour probabilities. This is a joint with Eugene Pechersky and Anatoly Yambartsev, both from IME-USP.

References

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