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On Laguerre-Hanh orthogonal polynomials on non-uniform lattices

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Resumo

Laguerre-Hahn orthogonal polynomials on non-uniform lattices were introduced by A.P. Magnus in [2]: a sequence of orthogonal polynomials is said to be Laguerre-Hahn if the corresponding formal Stieltjes function, S, satisfies a Riccati equation with polynomial coefficients

$$A(x)(\mathbb{D}S)(x) = B(x)(\mathbb{E}_1 S)(x)(\mathbb{E}_2 S)(x) + C(x)(\mathbb{M}S)(x) + D(x), \ A \neq 0, \quad (1)$$

where $\mathbb{D}$ is the divided difference operator involving the values of a function at two points, with the fundamental property that $\mathbb{D}$ leaves a polynomial of degree $n-1$ when applied to a polynomial of degree n [2, Eq. (1.1)]

$$(\mathbb{D}f)(x) = \frac{(\mathbb{E}_2 f)(x) - (\mathbb{E}_1 f)(x)}{y_2(x) - y_1(x)}, \quad (2)$$

with

$$(\mathbb{E}_1 f)(x) = f(y_1(x)), \quad (\mathbb{E}_2 f)(x) = f(y_2(x)).$$

In this talk it is given a characterization theorem for Laguerre-Hahn orthogonal polynomials on non-uniform lattices [1]. The theorem proves the equivalence between the Riccati equation for the formal Stieltjes function, linear first-order difference relations for the orthogonal polynomials as well as for the associated polynomials of the first kind, and linear first-order difference relations for the functions of the second kind.

Bibliografy

[1] A. Branquinho and M. N. Rebocho, Characterization theorem for Laguerre-Hahn orthogonal polynomials on non-uniform lattices, (submitted).

[2] A.P. Magnus, Associated Askey-Wilson polynomials as Laguerre-Hahn orthogonal polynomials, Springer Lect. Notes in Math. 1329, Springer, Berlin, 1988, pp. 261-278.