# Eigenvalue Decay of Positive Integral Operators

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**Resumo**

Let $\mathbb{M}$ be a compact two-point homogeneous space of dimension $m$. In this work, we will always consider $m \geq 2$. Let $dx$ be the usual volume element on $\mathbb{M}$ and $L^2(\mathbb{M})$ the Hilbert space of all square-integrable complex functions on $\mathbb{M}$ endowed with the usual inner product normalized $\langle f, g\rangle_2$ and the derived norm $||\cdot||_2$.

We will deal with integral operators defined by

$$\mathcal{K}(f) = \int_{\mathbb{M}} K(\cdot, y) f(y)\, dy, \tag{1}$$

in which the generating kernel $K\colon \mathbb{M}\times\mathbb{M} \to \mathbb{C}$ is an element of $L^2(\mathbb{M}\times\mathbb{M})$. In this case, (1) defines a compact operator on $L^2(\mathbb{M})$. If $K$ is $L^2$-*positive definite* in the sense that

$$\int_{\mathbb{M}}\int_{\mathbb{M}} K(x,y) f(x)\overline{f(y)}\, dxdy \geq 0, \quad f \in L^2(\mathbb{M}), \tag{2}$$

then $\mathcal{K}$ becomes a self-adjoint operator and the standard spectral theorem for compact and self-adjoint operators is applicable and we can write

$$\mathcal{K}(f) = \sum_{n=0}^{\infty} \lambda_n(\mathcal{K})\langle f, f_n\rangle_2 f_n, \quad f \in L^2(\mathbb{M}), \tag{3}$$

in which $\{\lambda_n(\mathcal{K})\}$ is a sequence of nonnegative reals (possibly finite) decreasing to 0 and $\{f_n\}$ is an $\langle\cdot,\cdot\rangle_2$-orthonormal basis of $L^2(\mathbb{M})$. The numbers $\lambda_n(\mathcal{K})$ are the eigenvalues of $\mathcal{K}$ and the sequence $\{\lambda_n(\mathcal{K})\}$ takes into account possible repetitions implied by the algebraic multiplicity of each eigenvalue.

We observe that the addition of continuity to $K$ implies that $\mathcal{K}$ is also *trace-class* (nuclear) ([2]). Consequently

$$\sum_{n=1}^{\infty} \lambda_n(\mathcal{K}) = \int_{\mathbb{M}} K(x,x)\, dx < \infty, \tag{4}$$

and we can extract the most elementary result on decay rates for the eigenvalues of such operators, namely,

$$\lambda_n(\mathcal{K}) = o(n^{-1}). \tag{5}$$

In this work we analyze the asymptotic behavior of the sequence $\{\lambda_n(\mathcal{K})\}$ under additional smoothness assumptions on the kernel $K$.

# Referências


[1] CASTRO, M. H.; MENEGATTO, V. A., *Eigenvalue decay of positive integral operators on the sphere.* Math. Comp. 81 (2012), no. 280, 2303-2317.

[2] GOHBERG, I. C.; KREIN, M. G. - *Introduction to the theory of linear nonselfadjoint operators.* Translations of Mathematical Monographs, Vol. 18 American Mathematical Society, Providence, R.I., 1969.