I Congresso Brasileiro de
Jovens Pesquisadores em
Matemática Pura e Aplicada

The Prism over Kneser Graphs is Hamiltonian

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Resumo

The vertices of the *Kneser graph* $K(n,k)$ are the k-subsets of $\{1,2,\ldots,n\}$ and two vertices are adjacent if the corresponding k-subsets are disjoint. For $n = 2k+1$, the Kneser graph $K(2k+1,k)$ is called the *odd graph* and it is denoted by O_k. The bipartite double graph of the Kneser graph $K(n,k)$ is known as the *bipartite Kneser graph* $B(n,k)$, whose vertices are the k-subsets, and $(n-k)$-subsets of $\{1,2,\ldots,n\}$ and the edges represent the inclusion between two such subsets. The graphs $K(n,k)$ and $B(n,k)$ are vertex-transitive and, therefore, they can provide a counterexample or more evidence to a long-standing conjecture due to Lovász which claims that every connected undirected vertex-transitive graph has a hamiltonian path.

It is well-known that the decision problem related to the hamiltonian cycle/path problem is NP-Complete. Thus, one recent trend is the search for related structures. In this aspect, having a hamiltonian prism in a graph was showed to be an interesting relaxation of being hamiltonian [3]. In fact, graphs having a hamiltonian prism are "closer" to being hamiltonian than graphs having a closed spanning walk where each vertex is traversed at most two times. The *prism over a graph* G is the Cartesian product $G\square K_2$ of G with the complete graph on two vertices. Previously, it was established that the prism over $B(2k+1,k)$ is hamiltonian [2]. Later, the counterpart of this result was proved for O_k but only for k even [1]. In our work, we show that the prism over the graphs $K(n,k)$ and $B(n,k)$ is hamiltonian for all $n > 2k$.

Referências

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